1 Practice (do not turn in)

Practice 1 A certain coin comes up heads with probability 0.6. If this coin is flipped three times, what is the probability of getting at least 2 heads?

Practice 2 People enter a certain store randomly at an average rate of 10 an hour. What is the probability that in a given hour, exactly 10 people enter?

Practice 3 In the previous problem, what is the probability that in a given hour, at least 6 people enter?

Practice 4 Births occur randomly in a certain town at a rate of 1 every 3 days. What is the probability that in the next 5 days, no one is born?

Practice 5 On any given stretch of road, there may be an accident anywhere, and the existence of an accident at one part of the road is independent of the existence of another accident at another part of the road. Suppose that for a given stretch of freeway, there is an average of .01 accidents per mile. Your commute involves 30 miles of this freeway. What is the probability that along your commute, there is at least one accident?

2 To Turn in

Problem 1 Which of the following random variables are Poisson? Binomial? Neither?

1. Of the next 10 telephone calls you make, how many of them are wrong numbers
2. How many landslides occur along PCH next year
3. How many Mondays next month PCH will be closed due to landslides
4. How many days in a row the stock market will go up starting from tomorrow
5. Of the next six days, on how many does the Dow Jones average go up
6. The result of rolling a fair die
7. The number of bankruptcies in the next ten years
Problem 2  You are applying for ten government contracts. The probability of each getting accepted is 0.2, and they are all independent. What is the probability of getting more than eight?

Problem 3  People arrive at a checkout counter randomly at an average rate of 2 per minute. What is the probability that in the next minute, there will be no one arriving at the counter?

Problem 4  People make a purchase on your website randomly at an average of 1 every 3 hours. What is the probability that in the next day, there will be at least 4 purchases on the website?

Problem 5  At a certain hospital’s maternity ward, children are born at random times, at an average of 3 every hour. If $X$ is the number of children born in the next hour, what is the expected value of $X$? The standard deviation of $X$?

Problem 6  Suppose $X$ is a Poisson random variable and the standard deviation of $X$ is 3. Find the probability that $X$ is less than 3.

3 Answers to practice

Practice 1  A certain coin comes up heads with probability 0.6. If this coin is flipped three times, what is the probability of getting at least 2 heads?

Solution 1  This is a binomial random variable, with $p = 0.6$ and $n = 3$. The probability of getting at least 2 heads is the probability of getting 2 heads, plus the probability of getting 3 heads:

\[
Pr(X \geq 2) = Pr(X = 2) + Pr(X = 3)
\]

\[
= C(3, 2)(.6)^2(.4)^1 + C(3, 3)(.6)^3(.4)^0
\]

\[
= .648
\]

Practice 2  People enter a certain store randomly at an average rate of 10 an hour. What is the probability that in a given hour, exactly 10 people enter?

Solution 2  Let $X$ be the number of people entering the store in that hour. Then $X$ is a Poisson random variable with $\lambda = 10$. We want to find the probability that $X = 10$, so we use the formula

\[
Pr(X = 10) = \frac{\lambda^{10}e^{-\lambda}}{10!}
\]

\[
= \frac{10^{10}e^{-10}}{10!}
\]

\[
\approx .1251
\]
Practice 3 In the previous problem, what is the probability that in a given hour, at least 6 people enter?

Solution 3 Again, if \( X \) is the number of people entering the store in that hour, \( X \) is Poisson with \( \lambda = 10 \). We want to find the probability that \( X \geq 6 \), which we can find by subtracting from 1 the probability that \( X < 6 \). That corresponds to the values 0, 1, 2, 3, 4, and 5:

\[
Pr(X < 6) = Pr(X = 0) + Pr(X = 1) + Pr(X = 2) + \cdots + Pr(X = 5)
\]
\[
= \frac{10^0}{0!}e^{-10} + \frac{10^1}{1!}e^{-10} + \cdots + \frac{10^5}{5!}e^{-10}
\]
\[
= \frac{10^0}{0!}e^{-10} + \frac{10^1}{1!}e^{-10} + \cdots + \frac{10^5}{5!}e^{-10}
\]
\[
\approx 0.0000454 + 0.000454 + 0.00227 + 0.00757 + 0.0189 + 0.0378
\]
\[
\approx 0.0671
\]

so the probability that \( X \) is at least 6 is \( 1 - 0.0671 = 0.9329 \).

Practice 4 Births occur randomly in a certain town at a rate of 1 every 3 days. What is the probability that in the next 5 days, no one is born?

Solution 4 Let \( X \) be the number of people born in the next 5 days. Births occur randomly at a rate of 1 every 3 days, or \( 1/3 \) per day. This turns into \( 5/3 \) every 5 days. So we see that \( X \) is Poisson with \( \lambda = 5/3 \).

\[
P(X = 0) = \frac{(5/3)^0}{0!}e^{-5/3}
\]
\[
= .1889
\]

Practice 5 On any given stretch of road, there may be an accident anywhere, and the existence of an accident at one part of the road is independent of the existence of another accident at another part of the road. Suppose that for a given stretch of freeway, there is an average of .01 accidents per mile. Your commute involves 30 miles of this freeway. What is the probability that along your commute, there is at least one accident?

Solution 5 The number of accidents along your commute is a Poisson random variable, with parameter \( \lambda = (30)(.01) = 0.3 \); The probability that there is at least one accident is 1 minus the probability of no accidents.

\[
Pr(X \geq 1) = 1 - p_{0.3}(0) = 1 - \frac{(0.3)^0}{0!}e^{-0.3} = 1 - e^{-0.3} \approx 0.259
\]