Problem 1 (5 pts) Take all the first-order partial derivatives of the following functions:

\[ f(x, y) = x^2 e^{xy} \]

\[ p(x, y) = xy + \ln y \]

Problem 2 (5 pts) Take all the first-order partial derivatives of the following functions:

\[ h(u, v) = \ln(u^2 v + 5) \]

\[ g(x, y, z) = xy^2 \]

Problem 3 (10 pts) Take all the first-order and second-order partial derivatives of the following function:

\[ U(x, t) = 10x^3 t^5 \]

Problem 4 (10 pts) The hardness \( H \) of a metal depends on the maximum temperature \( T \) in degrees Celsius during the production process and the purity \( P \). Suppose your metal production process has \( T = 8000 \) and \( P = 0.99 \). Also, suppose

\[ \frac{\partial H}{\partial T}(8000, 0.99) = 0.096 \]

and

\[ \frac{\partial H}{\partial P}(8000, 0.99) = 200. \]

Now suppose the temperature were to increase by 20 degrees Celsius, and the purity decreased by 0.01. What would be the approximate overall effect on the hardness?

Problem 5 (7 pts) Sketch the level set of \( f(x, y) = 2y + 4x \) at height 6.
Problem 6 (10 pts) Find all critical points of the function
\[ f(x, y) = 3x^2 - 3xy + 6y + 23 \]
and determine which are relative maxima, relative minima, and saddles.

Problem 7 (10 pts) True or false:

T F The partial derivative is so called because it is half a rate of change.

T F As you follow along a level curve for a function \( f(x, y) \), the value of \( f \) will not change.

T F If at a point \( (2, 4) \), \( f_x(2, 4) = 0 \), \( f_y(2, 4) = 0 \), and \( f_{xx}(2, 4) > 0 \), then we can conclude that \( (2, 4) \) is a relative minimum.

T F For a constrained optimization problem, at the maximum along a constraint, the level curve will be tangent to the constraint.

T F If \( p \) is the price of Ivory Soap, and \( q \) the price of Dove soap, and the demand for Ivory soap \( D(p, q) \) is viewed as a function of the price of Ivory and Dove, then we expect \( \frac{\partial D}{\partial q} > 0 \).

Problem 8 (8 pts) Suppose employee morale is a function \( M(p, q) \) where \( p \) is the pay rate and \( q \) is the desirability of the work they need to do. Interpret in words the following mathematical statement:

\[ \frac{\partial M}{\partial p} = 10 \]

Problem 9 (10 pts) Suppose a certain business produces two products: product A and product B. Let \( x \) represent the price of product A and \( y \) represent the price of product B, at which the business sells the product. Suppose the demand for product A is given by

\[ 1000 - x \]

and the demand for product B is given by

\[ 300 - y. \]

The cost of running the business is

\[ 600,000 - 500x - 100y. \]

Find the prices of the two products that maximize profit for the business.
Problem 10 (10 pts) Find the absolute maximum of $3x - 5y + 35$ subject to the constraint

$$x^2 + y^2 = 34.$$ 

Problem 11 (5 pts) The function $f(x, y)$ has a critical point at $(2, 7)$. At this critical point, $f_{xx}(2, 7) = 4$ and $f_{yy}(2, 7) = 30$. Give one example of a value for $f_{xy}(2, 7)$ for which $(2, 7)$ a saddle.

Problem 12 (10 pts) The function

$$f(x, y) = x^3 + x^2 + y^2 + x^2 y + xy$$

has a critical point at $(0, 0)$. Use the second derivative test to see what kind it is.