Problem 1 (10 pts) Find all first-order partial derivatives of the following functions:

\[ f(x, y) = 7x^{10}y + 2x + y^2 \]

\[ g(x, y, z) = xe^{xy} + yz \ln x \]

Problem 2 (10 pts) True or False:

T  F  The probability of an event is never greater than 1.

T  F  The odds in favor of an event is never greater than 1.

T  F  At a relative maximum that is not on the boundary of the domain, the first order partial derivatives of the function are zero.

T  F  When \( f(x, y) \) has continuous second derivatives, \( f_{xy} = f_{yx} \).

T  F  The probability of two things happening is found by multiplying the probability of the first thing times the probability of the second thing.

T  F  If \( A \) and \( B \) are sets, then \( (A \cup B)' = A' \cap B' \).

T  F  If \( E \) is any event, then \( Pr(E) = n(E)/n(S) \).

T  F  The standard deviation for any random variable is \( \sqrt{npq} \).

T  F  A system of linear equations has either 0, 1, or infinitely many solutions.

T  F  If a matrix has an inverse, it must be square.

Problem 3 (5 pts) Suppose you have a production function \( P(K, L) \) that is a function of capital \( K \) and labor \( L \). Suppose at the current values for capital and labor,

\[ \frac{\partial P}{\partial K} = 20, \quad \frac{\partial P}{\partial L} = 30. \]

What is the effect on production of increasing capital by 5 and not changing labor?
Problem 4 (5 pts) Sketch the level set of \( f(x, y) = x + y \) at level 2.

Problem 5 (10 pts) Find the critical points of
\[
x^2y - 3xy + 2y - 2x^2 + 5x + 10
\]
and determine which are maxima, minima, and saddles.

Problem 6 (10 pts) Use Lagrange multipliers to maximize
\[
2x - 5y
\]
subject to the constraint
\[
x^2 + 3y^2 = 444.
\]

Problem 7 (5 pts) Let
\[
A = \{ \text{students who are taking summer school this summer} \}
B = \{ \text{students who normally get financial aid} \}
C = \{ \text{students who have a car} \}
U = \{ \text{students} \}
\]
Write a mathematical expression involving \( A, B \) and/or \( C \), for the set of students not taking summer school this summer who either have a car or normally get financial aid (or both).

Problem 8 (5 pts) Draw a Venn diagram illustrating \( E \cup (F \cap G') \).

Problem 9 (5 pts) This summer you plan to take a trip to Japan. There are five possible flights you can choose from to get there, and seven possible return flights. If you are booking a flight there and a flight back, how many possible flight plans are you choosing from?

Problem 10 (5 pts) Acme Cuberat Inc. is hiring paper pushers. There are 12 applicants, and they will hire 3. How many possible outcomes are there as to who is hired?
Problem 11 (5 pts) There are seven workers at Fiefdom Corp., and they each have a desk. They now move to a new building that has 10 desks. How many possible assignments are there of a desk to each worker? Of course, there will be empty desks in such assignments.

Problem 12 (5 pts) There are ten raffle tickets: three winners and seven losers. If we choose four raffle tickets at random, what is the probability of getting two winners and two losers?

Problem 13 (5 pts) You are playing a game. If the probability of your winning is five times the probability of your losing, what is your probability of winning? Assume you either win or lose—there are no ties.

Problem 14 (5 pts) Suppose $E$, $F$, and $G$ are events. Suppose we have the following probability information:

\[
\begin{align*}
Pr(E) &= .4 \\
Pr(F) &= .4 \\
Pr(G) &= .2 \\
Pr(E \cap F) &= .2 \\
Pr(E \cap G) &= .1 \\
Pr(F \cap G) &= .1 \\
Pr(E \cap F \cap G) &= .1
\end{align*}
\]

Find $Pr(F' \cup G)$.

Problem 15 (10 pts) Timmy and Tommy are identical twins, and you can’t tell them apart. But when faced with a math 215 question, Timmy gets it right 60% of the time, and in such a situation, Tommy gets it right 70% of the time.

One day you see Timmy—or maybe it’s Tommy; you can never tell which. Come to think of it, it’s as likely to be Timmy as it is to be Tommy. He is working out a math 215 question and he gets it right. Given this information, what is the probability that it is Timmy?

Problem 16 (5 pts) Your company opens four stores. Their success is independent of the others. The first succeeds with a probability of .7. The second, with a probability of .9. The third, with a probability of .5, and the fourth, with a probability of .8. What is the probability that they all succeed?
Problem 17 (5 pts) A certain stock has a 20% probability of going up $10, a 50% probability of going up $5, and a 30% probability of going down $5. Find the standard deviation of the gain in the stock.

Problem 18 (10 pts) Indicate for each situation, whether $X$ is binomial, Poisson, normal, or neither.

<table>
<thead>
<tr>
<th>Binomial</th>
<th>Poisson</th>
<th>normal</th>
<th>neither</th>
<th>$X$ is either</th>
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</thead>
<tbody>
<tr>
<td>4 or 5, each with probability $1/2$.</td>
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<tbody>
<tr>
<td>You watch a certain gas station. $X$ is the number of cars that stop by in the next 15 minutes.</td>
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<tr>
<td>Of the next 5 phone calls you receive, $X$ is the number of these calls that are selling something.</td>
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<tr>
<td>You just bought a new printer. $X$ is the amount of time before you need to get a new ink cartridge.</td>
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<tr>
<td>Flip a fair coin until you get heads. $X$ is the number of times you had to flip the coin.</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Problem 19 (5 pts) You send your résumé to 12 companies, each independently with a 10% probability of giving you a job offer. What is the probability you get at least 3 job offers?

Problem 20 (5 pts) A new toy has come out, and its price is a normal random variable, with mean $30 and standard deviation $5. Find the probability that its price will be above $38.

Problem 21 (5 pts) For the same toy, find a price that is lower than 95% of the other prices.

Problem 22 (5 pts) You run a parking lot. Cars arrive randomly at an average rate of 1 every 5 minutes. What is the probability that in the next 7 minutes, exactly two cars arrive?
Problem 23 (5 pts) Below is a matrix representing a system of linear equations. We are in the middle of Gauss–Jordan elimination. Do the next step in Gauss–Jordan elimination only.

\[
\begin{bmatrix}
1 & 1 & 3 & 7 & 1 \\
0 & 0 & 2 & 3 & 2 \\
0 & 2 & 5 & 6 & 1 \\
\end{bmatrix}
\]

Problem 24 (5 pts) Same with this matrix: Do one step only.

\[
\begin{bmatrix}
1 & 0 & 5 & 5 & 6 \\
0 & 0 & 1 & 2 & 1 \\
0 & 0 & 3 & 6 & -1 \\
\end{bmatrix}
\]

Problem 25 (5 pts) Here is the result of doing Gauss–Jordan elimination. The columns correspond to variables \(x\), \(y\), and \(z\). Describe the solutions.

\[
\begin{bmatrix}
1 & 2 & 0 & 5 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 3 \\
\end{bmatrix}
\]

Problem 26 (10 pts) In each of the below situations, we have performed Gauss–Jordan elimination, and shown the resulting matrix. For each case, state how many solutions there are.

- \[
\begin{bmatrix}
1 & 3 & 0 & 4 \\
0 & 0 & 1 & 5 \\
\end{bmatrix}
\]
- \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 2 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
- \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
- \[
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 2 \\
0 & 0 & 0 & 2 \\
\end{bmatrix}
\]
- \[
\begin{bmatrix}
0 & 1 & 0 & 4 \\
0 & 0 & 1 & 5 \\
\end{bmatrix}
\]
Problem 27 (5 pts) Here is the result of doing Gauss–Jordan elimination. The columns correspond to variables \(a, b, c,\) and \(d\). Find the solutions.
\[
\begin{bmatrix}
1 & 1 & 0 & 3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Problem 28 (5 pts) Do the same for this matrix (columns are again \(a, b, c,\) and \(d\)):
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

Problem 29 (5 pts) Suppose we have a Leontieff input-output model with \(A\) matrix:
\[
A = \begin{bmatrix}
0.3 & 0.1 \\
0.2 & 0.1
\end{bmatrix}
\]
and outside demand
\[
D = \begin{bmatrix}
120 \\
200
\end{bmatrix}
\]
Find the predicted production matrix.

Problem 30 (5 pts) Perform the following matrix multiplication:
\[
\begin{bmatrix}
2 & 1 \\
5 & 0 \\
4 & 4
\end{bmatrix}
\begin{bmatrix}
7 \\
8
\end{bmatrix}
\]

Problem 31 (5 pts) Find the inverse of
\[
\begin{bmatrix}
3 & 7 \\
2 & 4
\end{bmatrix}
\]

Problem 32 (5 pts) Use the fact that the inverse of
\[
\begin{bmatrix}
12 & -3 & 4 & -5 & 6 & -4 & 2 & -3 \\
-23 & 6 & -8 & 10 & -12 & 8 & -4 & 6 \\
34 & -8 & 12 & -15 & 18 & -12 & 6 & -9 \\
-45 & 10 & -15 & 20 & -2 & 16 & -8 & 12 \\
54 & -12 & 18 & -24 & 30 & -20 & 10 & -15 \\
-36 & 8 & -12 & 16 & -20 & 14 & -7 & 10 \\
18 & -4 & 6 & -8 & 10 & -7 & 4 & -5 \\
-27 & 6 & -9 & 12 & -15 & 10 & -5 & 8
\end{bmatrix}
\]
is

\[
\begin{bmatrix}
  2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\
4 & 2 & 1 & 2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 \\
\end{bmatrix}
\]

to solve the following system of linear equations:

\[
\begin{align*}
12a - 3b + 4c - 5d + 6e - 4f + 2g - 3h &= 0 \\
-23a + 6b - 8c + 10d - 12e + 8f - 4g + 6h &= 0 \\
34a - 8b + 12c - 15d + 18e - 12f + 6g - 9h &= 3 \\
-45a + 10b - 15c + 20d - 2e + 16f - 8g + 12h &= 0 \\
54a - 12b + 18c - 24d + 30e - 20f + 10g - 15h &= 0 \\
-36a + 8b - 12c + 16d - 20e + 14f - 7g + 10h &= 0 \\
18a - 4b + 6c - 8d + 10e - 7f + 4g - 5h &= 0 \\
-27a + 6b - 9c + 12d - 15e + 10f - 5g + 8h &= 1
\end{align*}
\]

**Problem 33 (5 pts)** Solve the following matrix equation for \(X\):

\[
\begin{bmatrix}
  1 & 3 \\
 4 & 13
\end{bmatrix}
X
\begin{bmatrix}
  3 & 2 \\
 3 & 3
\end{bmatrix} +
\begin{bmatrix}
  1 & 1 \\
 4 & 3
\end{bmatrix} =
\begin{bmatrix}
  0 & 2 \\
 6 & 6
\end{bmatrix}
\]

**Problem 34 (5 pts)** Write down an example of a matrix \(A\) and a matrix \(B\) for which \(AB \neq BA\).