Math 215: Derivatives Review

The following derivatives should be memorized. Here $x$ is the independent variable, and $c$ is any constant.

\[
\frac{d}{dx}c = 0
\]

\[
\frac{d}{dx}x^n = nx^{n-1}
\]

\[
\frac{d}{dx}e^x = e^x
\]

\[
\frac{d}{dx}a^x = (\ln a)(a^x)
\]

\[
\frac{d}{dx}\ln x = \frac{1}{x}
\]

The following rules should be memorized:

**Rule 1 (Addition Rule)**

\[
\frac{d}{dx}(f + g) = \frac{d}{dx}f + \frac{d}{dx}g
\]

**Example 1**

\[
\frac{d}{dx}(x^3 + \ln x) = 3x^2 + \frac{1}{x}
\]

**Rule 2 (Subtraction Rule)**

\[
\frac{d}{dx}(f - g) = \frac{d}{dx}f - \frac{d}{dx}g
\]

**Example 2**

\[
\frac{d}{dx}(x^3 - \ln x) = 3x^2 - \frac{1}{x}
\]

**Rule 3 (Multiplying by a constant)**

\[
\frac{d}{dx}(cf) = c\frac{d}{dx}f
\]

**Example 3**

\[
\frac{d}{dx}(8x^3) = 24x^2
\]
Rule 4 (Product Rule)

\[ \frac{d}{dx}(fg) = f \frac{d}{dx}g + g \frac{d}{dx}f \]

Example 4

\[ \frac{d}{dx}(x^3e^x) = x^3e^x + e^x(3x^2) \]

Rule 5 (Quotient Rule)

\[ \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g \frac{d}{dx}f - f \frac{d}{dx}g}{g^2} \]

Example 5

\[ \frac{d}{dx}\left(\frac{x^3}{e^x}\right) = \frac{e^x(3x^2) - x^3e^x}{(e^x)^2} \]

Note that this is not strictly speaking necessary, since you can view \( f/g \) as \( f \cdot g^{-1} \), and use the product rule and the chain rule together, like this:

\[ \frac{d}{dx}\left(\frac{x^3}{e^x}\right) = \frac{d}{dx}x^3(e^{-x})^{-1} = 3x^2(e^{-x})^{-1} + x^3(-1)(e^{-x})^{-2}(e^{-x}) \]

Rule 6 (Chain Rule)

\[ \frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \]

Example 6

\[ \frac{d}{dx}\ln(x) = \frac{1}{x} \]

- When dealing with \( \sqrt{x} \) or \( \sqrt[n]{x} \), turn them into \( x^{1/2} \) or \( x^{1/n} \) before differentiating.
When differentiating a function that is in the above table, and it has, instead of $x$, some other function of $x$, remember to use the chain rule and multiply by the derivative of the argument:

$$\frac{d}{dx}(e^{\sqrt{x}}) = e^{\sqrt{x}} \frac{d}{dx}\sqrt{x}$$

$$= e^{\sqrt{x}} \frac{d}{dx}x^{1/2}$$

$$= e^{\sqrt{x}} \frac{1}{2}x^{-1/2}$$