Math 510 HW #20
Due 11/18

Problem 1 Let $X_1, \ldots, X_n$ be independent, identically distributed (i.i.d.) random variables, each with expected value 3 and standard deviation 7.

Find the expected value and standard deviation of $\overline{X} = \frac{X_1 + \cdots + X_n}{n}$.

Problem 2 In the previous problem, find the value of $n$ so that the standard deviation of $\overline{X}$ is less than .001.

Problem 3 Let $X_1, \ldots, X_n$ be independent, identically distributed (i.i.d.) random variables, each with expected value $\mu$ and standard deviation $\sigma$. Take the sum $X_1 + \cdots + X_n$ and use it, together with $\mu$ and $\sigma$, to create a random variable with expected value 0 and standard deviation 1.

Problem 4 Use the law of large numbers (average form) to prove the law of large numbers (probability form).

Problem 5 Suppose you have a die, and $p_1, \ldots, p_6$ are defined with $p_k$ equal the probability of rolling $k$. Roll the die $n$ times. Let $X_1, \ldots, X_n$ be the rolls of the die. Using the law of large numbers (probability form), prove the law of large numbers (average form) for $X_1, \ldots, X_n$.

Problem 6 Let $X$ be a non-negative random variable, with expected value 10. Use Markov’s inequality to determine an inequality related to $P(X > 30)$. Use Markov’s inequality to find an interval for $X$ that is guaranteed to contain 99% of the probability.

Problem 7 Let $X$ be a random variable, with mean 100 and standard deviation 4. Use Chebyshev’s inequality to determine an inequality for $P(92 < X < 108)$. Use Chebyshev’s inequality to find an interval for $X$ that is guaranteed to have 99% of the probability.

Problem 8 Ch. 8 problems p. 457 #1, 2