

Math Day

November 14, 2002

PROBE I

EXAMINATION RULES AND INSTRUCTIONS

DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY THE PROCTOR.

1. Pull the ANSWER FORM out of your test booklet and provide the following information:

Name
Social Security Number
Sex
School CEEB number (Use Box A)

In each case, write the correct information in the boxes and then darken the corresponding circle under the box.

2. On the back of the ANSWER FORM use the upper left-hand corner to print the name of your school and the city in which it is located.
3. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D, and E. Only one answer is correct. You will use the first 25 spots on the answer form to record your answers.
4. **SCORING RULES:** You receive 4 points for each correct answer, 1 point for each question left unanswered and 0 points for each incorrect answer. As a result, random guessing will hurt your score but if you can eliminate at least two possible answers on a question, it is to your advantage to guess.
5. Use a number 2 pencil to record your answer. Calculators are **NOT** permitted.
6. Figures are not necessarily drawn to scale.
7. Do not open the test booklet until the proctor gives the signal to begin the test.
THE TEST WILL LAST 60 MINUTES.

1. Suppose $x_0 = 1114$ and for any $n \geq 0$, we have $x_{n+1} = 1 - \frac{1}{x_n}$. What is x_{2002} ?

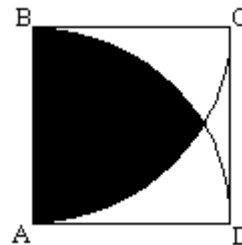
- A) $\frac{1113}{1114}$ B) $\frac{-1}{1113}$ C) 1114 D) 1113 E) 0

2. Find all real numbers p such that $px^2 + px - 1 \leq 0$ for all x .

- A) $p \geq 0$ B) $p \leq 0$ C) $p = 0$
 D) $-4 \leq p \leq 0$ E) $0 \leq p \leq 4$

3. What is the area of the shaded region below? Assume the square ABCD has side length 6 and the two circular arcs are centered at A and B and each has radius 6.

- A) $32\pi - 64$ B) $18\pi - 36$ C) $36\pi - 36$
 D) $6\pi - 6$ E) $12\pi - 9\sqrt{3}$



4. The exam average for a class of 25 students was 75. Later, 5 students received 10 bonus points each. What is the new class average on the exam?

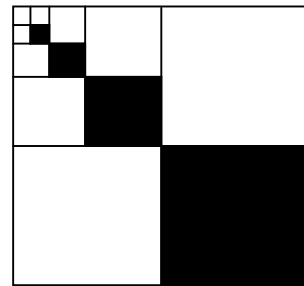
- A) 76 B) 77 C) 78 D) 79 E) 80

5. A circle of radius 8 is centered at the origin and a circle of radius 10 is centered at $(0, 12)$. What is the y coordinate of the point in the first quadrant at which they intersect?

- A) 3.5 B) 4 C) 4.5 D) 5 E) 5.5

16. What is the sum of the areas of the (infinitely many) shaded squares in the figure below? Assume the large square has side length 12, each subdivision divides a square into 4 equally sized smaller square and the process continues forever.

A) 18 B) 48 C) 72
D) 36 E) 81



17. In your bedroom you have a large supply of pennies, nickels, dimes, and quarters in a jar. What is the fewest total number of coins you would need to take from the jar to be able to make change anywhere from 1 cent to 99 cents?

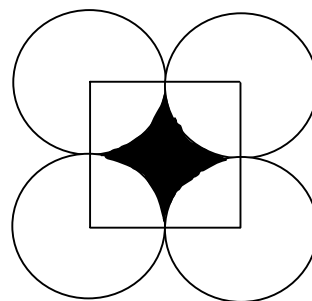
A) 9 B) 10 C) 11 D) 12 E) 13

18. Twenty men did one-fourth of a job in eight days. How many additional men must be added in order to finish the job in 5 more days?

A) 58 B) 62 C) 68 D) 72 E) 76

19. Four circles of radius 3 are arranged as shown and their centers form the vertices of a square. What is the area of the shaded region?

A) $36 - 9\pi$ B) $18 + \pi$ C) $36 + 4\pi$
D) $18 + 9\pi$ E) $9 + 4\pi$



20. Suppose a square of side length s is stretched into a rectangle by lengthening one pair of opposite sides by 12 inches and the other pair of opposite sides by 20 inches. If the area of the resulting rectangle is 513, what is the remainder of s upon division by 5?

A) 0 B) 1 C) 2 D) 3 E) 4

21. Which of the following is the largest?

- A) 1^{400} B) 2^{100} C) 3^{75} D) 5^{50} E) 22^{25}

22. If the perimeter of a rectangle is 100, what is the shortest diagonal the rectangle could have?

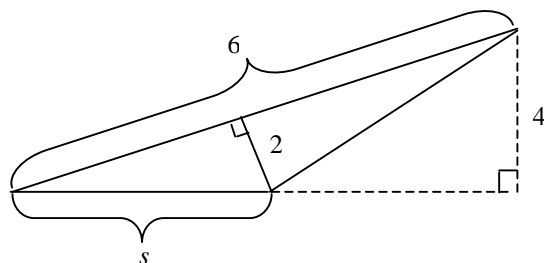
- A) 10 B) $10\sqrt{2}$ C) 25 D) $25\sqrt{2}$ E) 50

23. A bug starts at the origin and crawls 1 unit east, then $\frac{1}{2}$ unit north, then $\frac{1}{4}$ unit west, then $\frac{1}{8}$ unit south, then $\frac{1}{16}$ unit east, and so on forever. What are the coordinates of the bug's ultimate destination?

- A) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}})$ B) $(\frac{2}{3}, \frac{1}{3})$ C) $(\frac{3}{4}, \frac{3}{8})$ D) $(\frac{1}{2}, \frac{1}{2})$ E) $(\frac{4}{5}, \frac{2}{5})$

24. What is the value of s in the following diagram?

- A) 4 B) $4\sqrt{5} - 3$ C) $2\sqrt{5}$
D) 3 E) $4\sqrt{5}$



25. A dog is tied to the corner of a ten-foot by 20-foot shed on a rope having length 60 feet. Assume the dog starts out as pictured below and winds his way around the shed counter-clockwise as far as he can go. What is the total area (in square feet) swept out by the rope?

- A) 7400π B) 3600π C) 1800π
D) 1850π E) 900π

