Math 215  Spring 2010  Midterm Exam 1  February 2, 2010

Name: _____________________________________________________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>1/2</th>
<th>3</th>
<th>4/5</th>
<th>6</th>
<th>7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible</td>
<td>25</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td>25</td>
<td>100</td>
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<td>Received</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.
You may use one page (one side) of notes, but no other materials or resources (such as notes, old HW, etc.).
There is no sharing with a friend or neighbor.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.
1. Answer each of the following questions. No explanation is needed.

T F Some systems $AX = B$ of linear equations have an infinite number of solutions.

T F The inverse of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

T F One solution to $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 0 \\ 1 & 0 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is $\begin{bmatrix} 6 \\ 11 \\ 21 \end{bmatrix}$.

T F For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $AX = B$ has a solution $X$ regardless of the right hand side $B$.

T F A system of 3 equations and 2 unknowns (variables) cannot have a unique solution.

2. Find the $2 \times 2$ matrix $C$ for which $A \cdot C = B$, where $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$.
3. A company produces two items, but uses up some of each product in the production process, as described by the input-output matrix

\[ A = \begin{bmatrix} .3 & .1 \\ .1 & .7 \end{bmatrix}. \]

In working this problem, one of the following inverses will be useful to you:

\[ \begin{bmatrix} .3 & .1 \\ .1 & .7 \end{bmatrix}^{-1} = \begin{bmatrix} 3.5 & -.5 \\ -.5 & 1.5 \end{bmatrix} \quad \begin{bmatrix} .3 & -.1 \\ -.1 & .7 \end{bmatrix}^{-1} = \begin{bmatrix} 3.5 & .5 \\ .5 & 1.5 \end{bmatrix} \]

\[ \begin{bmatrix} .7 & .1 \\ .1 & .3 \end{bmatrix}^{-1} = \begin{bmatrix} 1.5 & -.5 \\ -.5 & 3.5 \end{bmatrix} \quad \begin{bmatrix} .7 & -.1 \\ -.1 & .3 \end{bmatrix}^{-1} = \begin{bmatrix} 1.5 & .5 \\ .5 & 3.5 \end{bmatrix} \]

\[ \begin{bmatrix} .7 & .9 \\ .9 & .3 \end{bmatrix}^{-1} = \begin{bmatrix} -.5 & 1.5 \\ 1.5 & -1.17 \end{bmatrix} \quad \begin{bmatrix} .7 & -.9 \\ -.9 & .3 \end{bmatrix}^{-1} = \begin{bmatrix} -.5 & -1.5 \\ -1.5 & -1.17 \end{bmatrix} \]

(a) If we produce 10 units of each product, how much is used up in the production process, and thus how much do we actually end up with?

(b) How many units of each product should be produced in order to end up with 10 units of each product?
4. We are interested in solving the following system of equations,

\[
\begin{align*}
2x + 3y &= a \\
4x + by &= 8
\end{align*}
\]

where \( a \) and \( b \) are some constants (whose values haven’t yet been decided). Give an example of values of \( a \) and \( b \) that result in the system having:

- No solution: \( a = \) \( b = \)
- One solution: \( a = \) \( b = \)
- Infinite solutions: \( a = \) \( b = \)

5. Use the Gauss-Jordan method to find the inverse of

\[
\begin{bmatrix}
1 & 2 & -2 \\
1 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}
\]
6. A 350-seat movie theater charges $8.50 admission for adults and $5.50 for children. If the theater is full and $2711 is collected, how many adults and how many children are in the audience? (Hints: $8.5(350) = 2975$ and $264/3 = 88$.)
7. Find all solutions to each of the following linear systems:

\[
\begin{align*}
  x + 2y - z + 3w &= 5 \\
  y + 2z + w &= 7 \\
\end{align*}
\]

\[
\begin{align*}
  x + y &= -6 \\
  -2x + 3y &= 17 \\
\end{align*}
\]

\[
\begin{align*}
  p + n &= 6 \\
  p - 5n &= 0 \\
  p + 5n &= 10 \\
\end{align*}
\]