Approximate grade breakdown (“if I curved this exam”):

- 90 – 100 A
- 80 – 90 B
- 0 – 80 C or below

Page 2

3 \cdot 8 \cdot 9

\binom{7}{3}

\binom{100}{40} \quad \text{Once you decide which 40 Aaron gets, the other 60 go to Brian, so you are really only choosing which 40 from 100 to give to Aaron. If you insist on “choosing” for Brian as well, then the answer would be } \binom{100}{40} \cdot \binom{60}{60}, \text{ which is simply } \binom{100}{40}, \text{ since you get to choose 60 for Brian from the 60 that remain once you’ve chosen the 40 for Aaron.}

\binom{5}{3} \cdot 9 \cdot 9 \cdot 8 \quad \text{You first choose which 3 of the 5 digits will be the same, then choose the first digit from 1 to 9, then the next digit from 0 to 9 (minus the one chosen for the first digit), then the final digit from 0 to 9 (minus the two already used).}

26 \cdot 26 \cdot 26 \cdot 26

\binom{13}{3} \text{ or } \binom{13}{10}, \text{ like the examples we saw from the book and in class.}

Page 3

2^6, \text{ or the long way of } \binom{6}{0} + \binom{6}{1} + \cdots + \binom{6}{6}

6 \cdot 2

P(7,3) = 7 \cdot 6 \cdot 5, \text{ or } \binom{7}{3} \cdot 3!

\binom{50}{10} \cdot \binom{40}{15} \cdot \binom{25}{25} = \frac{50!}{10! \cdot 15! \cdot 25!}

10!

Since everyone will get a book, the question really is how many ways to distribute 3 books to 1 or more persons, which is \binom{10}{1} + P(10,2) + \binom{10}{3}:

- There are \binom{10}{1} ways to choose which one person could get all 3.
- There are \binom{10}{2} \cdot 2! ways to choose which two persons of whom 1 would get 1 book and the other would get 2 books.
- There are \binom{10}{3} ways to choose which three persons would each get 1.
Since 1! = 1, 2! = 2, 3! = 6, 4! = 24, and 5! = 120, there must be 5 books to order.

\[ C(20,2) = \frac{20!}{2!18!} = \frac{20 \cdot 19 \cdot 18!}{2 \cdot 18!} = 10 \cdot 19 = 190. \]

\[ P(6,3) = 6 \cdot 5 \cdot 4 = 120. \]

4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24.

\[ C(50,7) \]

# ways to choose the 2 9's * # ways to choose any 5 of the other 45 cards =

\[ C(5,2) \cdot C(45,5) \]

\[ 4 \cdot C(5,1) \cdot C(5,1) \cdot C(5,1) \cdot C(5,1) \cdot C(5,1) \cdot C(5,1) \cdot C(5,1) = 4 \cdot 5^7 \]

There are 4 possible straights: 1 – 7; 2 – 8; 3 – 9; 4 – 10; for each of the 7 numbers in the straight, there are 5 choices for which color of that number.

Number of ways to choose which 2 numbers (order matters: whichever you choose first you will have four of) * number of ways to choose the 4 from 5 * number of ways to choose the 3 from the 5 =

\[ P(10,2) \cdot C(5,4) \cdot C(5,3). \]

\[ C(100,5) \]

\[ C(50,5) \cdot 2^5, \text{ or } \frac{100 \cdot 98 \cdot 96 \cdot 94 \cdot 92}{5!} \]

There are 50 women total, so \( C(50,5) \).

Choose 2 from 50 men, and 3 from 50 women, so \( C(50,2) \cdot C(50,3) \).

Choose which 50 states (so you now have 5 men from each state and 5 women from each state), then from 2 of the states the two men (which also means it is automatically determined which 3 women), so \( C(50,5) \cdot C(5,2) \);

OR

Choose 2 men from the 50 total men, then choose the 3 women from the 48 that come from the 48 states different than those the 2 chosen men come from, so \( C(50,2) \cdot C(48,3) \).

Of course \( C(50,5) \cdot C(5,2) = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{2 \cdot 3!} = C(50,2) \cdot C(48,3) \).