Approximate grade breakdown (“if I curved this exam”):

- 90 – 100 A
- 80 – 90 B
- 0 – 80 C or below

1. 

![Venn Diagram](image)

\[
\Pr(E \cup F) = \frac{5}{7} \quad \text{from the diagram, or} \quad \Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F) = \frac{3}{7} + \frac{4}{7} - \frac{2}{7} = \frac{5}{7}.
\]

\[
\Pr(E') = 1 - \Pr(E) = 1 - \frac{3}{7} = \frac{4}{7}.
\]

\[
\Pr(E' \cap F) = \frac{2}{7} \quad \text{from the diagram, or} \quad \Pr(E' \cap F) = \Pr(F) - \Pr(E \cap F) = \frac{4}{7} - \frac{2}{7} = \frac{2}{7}.
\]

\[
\Pr(E \mid F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{2/7}{4/7} = \frac{2}{4}.
\]

\[
\Pr(F \mid E) = \frac{\Pr(E \cap F)}{\Pr(E)} = \frac{2/7}{3/7} = \frac{2}{3}.
\]

2. Next page.

3. \(\Pr(\text{Odd}) = \frac{5}{6}\)

\[
\Pr(\square \text{Odd}) = \frac{3}{5}
\]

\[
\Pr(3 \text{ and 3 and 3 and 3 and 3}) = \Pr(3) \cdot \Pr(3) \cdot \Pr(3) \cdot \Pr(3) \cdot \Pr(3) = \left(\frac{2}{6}\right)^5 = \left(\frac{1}{3}\right)^5
\]

\[
\Pr(\text{at least one 1 or 2}) = 1 - \Pr(\text{not "at least one 1 or 2"}) = 1 - \Pr(\text{all 3's}) = 1 - \left(\frac{1}{3}\right)^5
\]
2.

<table>
<thead>
<tr>
<th>Probability</th>
<th>If no testing is done</th>
<th>If test is positive</th>
<th>If test is negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has flu</td>
<td>.0500</td>
<td>.6225</td>
<td>.0032</td>
</tr>
<tr>
<td>Does not have flu</td>
<td>.9500</td>
<td>.3775</td>
<td>.9968</td>
</tr>
</tbody>
</table>

Where F is “has flu” and + is “tests positive,” from the given information:

\[
\begin{align*}
\Pr(F) &= .05 & \Pr(+ \mid F) &= .94 & \Pr(+ \mid F') &= .03 \\
\Pr(F') &= .95 & \Pr(- \mid F) &= .06 & \Pr(- \mid F') &= .97
\end{align*}
\]

We first find that

\[
\Pr(+) = \Pr(\{+ \cap F\} \cup \{+ \cap F'\}) = \Pr(+ \cap F) + \Pr(+ \cap F')
\]

\[
= \Pr(F) \Pr(+ \mid F) + \Pr(F') \Pr(+ \mid F') = (.05)(.94) + (.95)(.03) = .0755
\]

and consequently \(\Pr(-) = 1 - \Pr(+) = 1 - .0755 = 0.9245\), or you could (unnecessarily) do all the work to find

\[
\Pr(-) = \Pr(\{- \cap F\} \cup \{- \cap F'\}) = \Pr(- \cap F) + \Pr(- \cap F')
\]

\[
= \Pr(F) \Pr(- \mid F) + \Pr(F') \Pr(- \mid F') = (.05)(.06) + (.95)(.97) = .9245
\]

Next, if you test positive, then

\[
\Pr(F \mid +) = \frac{\Pr(F \cap +)}{\Pr(+)} = \frac{\Pr(F) \Pr(+ \mid F)}{\Pr(+)} = \frac{(.05)(.94)}{.0755} \approx .6225,
\]

and consequently

\[
\Pr(F' \mid +) = 1 - \Pr(F \mid +) \approx 1 - .6225 = .3775
\]

or you could have done all the work to find that

\[
\Pr(F' \mid +) = \frac{\Pr(F' \cap +)}{\Pr(+)} = \frac{\Pr(F') \Pr(+ \mid F')}{\Pr(+)} = \frac{(.95)(.03)}{.0755} \approx .3775
\]

Finally, if you test negative, then

\[
\Pr(F \mid -) = \frac{\Pr(F \cap -)}{\Pr(-)} = \frac{\Pr(F) \Pr(- \mid F)}{\Pr(-)} = \frac{(.05)(.06)}{.9245} \approx .0032,
\]

and \(\Pr(F' \mid -) = 1 - \Pr(F' \mid +) = 1 - .0047 = .9968\)

or you could have done

\[
\Pr(F' \mid -) = \frac{\Pr(F' \cap -)}{\Pr(-)} = \frac{\Pr(F') \Pr(- \mid F')}{{\Pr(-)}} = \frac{(.95)(.97)}{.9245} = .9968
\]
4. Where:  
\( R1 \) means the first ball chosen is red  
\( R2 \) means the second ball chosen is red  
\( B1 \) means the first ball chosen is blue  
\( B2 \) means the second ball chosen is blue

Since \( \Pr(R1) = \frac{3}{10} \),
then \( \Pr(R1 \cap B2) = \Pr(R1) \cdot \Pr(B2 \mid R1) = \frac{3}{10} \cdot \frac{7}{9} = \frac{21}{90} \), and

\[
\Pr(B2) = \Pr((R1 \cap B2) \cup (B1 \cap B2)) = \Pr(R1 \cap B2) + \Pr(B1 \cap B2) = \frac{3}{10} \cdot \frac{7}{9} + \frac{7}{10} \cdot \frac{6}{9} = \frac{63}{90} = \frac{7}{10}
\]

(I wanted more than an intuitive answer, since sometimes our intuition leads us to wrong answers—by the way, I actually did this in class one day)

so that \( \Pr(R1 \cup B2) = \Pr(R1) + \Pr(B2) - \Pr(R1 \cap B2) = \frac{3}{10} + \frac{7}{10} - \frac{21}{90} = 1 - \frac{21}{90} = \frac{69}{90} \).

Or you could have also consider the three ways this could occur to find that

\[
\Pr(R1 \cup B2) = \frac{3}{10} \cdot \frac{2}{9} + \frac{3}{10} \cdot \frac{7}{9} + \frac{7}{10} \cdot \frac{6}{9} = \frac{69}{90}
\]

Next, \( \Pr(R2 \mid B1) = \frac{3}{9} \), since all 3 red balls remain of the 9 total (after choosing one blue one), or you could show that \( \Pr(R2 \mid B1) = \frac{\Pr(B1 \cap R2)}{\Pr(B1)} = \frac{(7/10)(3/9)}{7/10} = \frac{3}{9} \).

Finally, \( \Pr(R1 \mid B2) = \frac{\Pr(R1 \cap B2)}{\Pr(B2)} = \frac{21/90}{7/10} = \frac{21}{90} \cdot \frac{10}{7} = \frac{3}{9} \).

Your intuition tells you that the two events \( R1 \) and \( B2 \) are not independent, since knowing whether the first ball is red will affect the likelihood that the second ball is blue. Proof: they are independent if \( \Pr(R1 \cap B2) = \Pr(R1) \Pr(B2) \), and dependent otherwise. Since \( \Pr(R1 \cap B2) = \frac{21}{90} \neq \frac{3}{10} \cdot \frac{7}{10} = \Pr(R1) \Pr(B2) \), they are not independent. You could also show they are not independent by pointing out that \( \Pr(R1 \mid B2) \neq \Pr(R1) \).