Problem 1 2 3 Total
Possible 20 10 70 100
Received

**DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.**
You may use one page (one side) of notes, but no other materials or resources (such as a calculator, notes, old HW, etc.). **There is no sharing with a friend or neighbor.**

**FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.**
1. Some set questions:

Shade in the set $(R \cap S \cap T^c) \cup T$.

What set does the following diagram represent?

One hundred college students were surveyed after voting in an election involving a Democrat and a Republican. 50 were first-year students, 55 voted Democrat, and 25 were non-first-year students who voted Republican. How many first-year students voted Democratic?

Simplify (in terms of A, B and/or C):

$$(A')' =$$

$$A \cap B \cap A' =$$

$$(A \cap B \cap C')' =$$

$$(A \cup B \cup A')' =$$
2. The 100 members of a pet club were asked what pets they own. Their responses:
   - 35 own a dog
   - 40 own a cat
   - 25 own a fish
   - 20 own a dog and a cat
   - 15 own a dog and a fish
   - 10 own a cat and a fish
   - 5 own a dog, cat and a fish

Draw a Venn Diagram corresponding to this information, and use it to answer the subsequent questions.

How many own no pets?

How many own a cat or a fish (but not both), and not a dog?

How many own exactly one pet?
3. In answering the following questions, except where noted, do not simplify the answers. For example, leave your answer in the form \( \binom{5}{3} \) or \( 12! \) or \( P(4,3) \cdot P(7,4) \) or \( 2^5 - 2^3 \) or \( 7 \cdot 6 \cdot 5 \cdot 4 \).

Five friends share an apartment. Some (or none or all) of them are getting together for dinner this evening. How many different combinations of those friends could there be at dinner?

In how many ways can 3 people be assigned to seats in a 6-seat room?

How many 10-digit phone numbers (3-digit area code plus 7-digit number) are there? Assume that you cannot have a 0 as the first digit of the area code or as the first digit of the phone number.

In how many different ways can 14 different boxes of girl scout cookies be distributed to 14 of your friends?

How many different meals can be chosen if there are 5 appetizers, 9 main dishes, and 11 desserts, assuming a meal consists of one item from each category?

In how many 4-digit numbers (i.e. 1000 – 9999) are there in which 2 or more of the digits are alike?

In how many ways can a committee of 5 persons be chosen from 12 married couples if:

The committee must consist of 2 men and 3 women?

A couple (a husband and his wife) cannot both serve on the committee?
(Problem 3 continued)

Suppose there are 25 athletes competing in an Olympic event. How many different outcomes (in terms of who wins what prize) are there if:

/4 There are 3 winners, with a different prize for each (e.g. Gold, Silver, Bronze), according to how each did in the event.

/4 There are 3 winners, with the same prize for each.

/4 All 25 win, with a different prize for each, according to how each did in the event.

/4 All 25 win, with the same prize for each.

For the next five questions, suppose there are three boys and three girls at a party. Simplify these next five answers, i.e. give actual numbers (and show your work).

/4 In how many different ways could we choose one couple (one boy, one girl)?

/4 In how many different ways could we choose any two of them (of either gender)?

/4 In how many different ways could we divide the six into three boy-girl couples?

/4 In how many ways can they be seated in a row such that no person is seated next to someone of the same gender?

/4 In how many different ways could we choose a president and a vice-president from these six kids?