DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.
You may use one page (one side) of notes, but no other materials or resources (such as a calculator notes, old HW, etc.).
There is no sharing with a friend or neighbor.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.
1. Find the inverse of $\begin{bmatrix}
1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 \\
3 & 2 & 1 & 0 \\
4 & 3 & 2 & 1 \\
\end{bmatrix}$. 

/15
2. A company produces two items, but uses up some of each product in the production process, as described by the input-output matrix

\[
A = \begin{bmatrix}
.3 & .1 \\
.1 & .7 \\
\end{bmatrix}
\]

In working this problem, one of the following inverses will be useful to you:

\[
\begin{bmatrix}
.3 & .1 \\
.1 & .7 \\
\end{bmatrix}^{-1} = \begin{bmatrix}
3.5 & -.5 \\
-.5 & 1.5 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.3 & -.1 \\
-.1 & .7 \\
\end{bmatrix}^{-1} = \begin{bmatrix}
3.5 & .5 \\
.5 & 1.5 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.7 & .1 \\
.1 & .3 \\
\end{bmatrix}^{-1} = \begin{bmatrix}
1.5 & -.5 \\
-.5 & 3.5 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.7 & -.1 \\
-.1 & .3 \\
\end{bmatrix}^{-1} = \begin{bmatrix}
1.5 & .5 \\
.5 & 3.5 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.7 & .9 \\
.9 & .3 \\
\end{bmatrix}^{-1} = \begin{bmatrix}
-.5 & 1.5 \\
1.5 & -1.17 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.7 & -.9 \\
-.9 & .3 \\
\end{bmatrix}^{-1} = \begin{bmatrix}
-.5 & -1.5 \\
-1.5 & -1.17 \\
\end{bmatrix}
\]

(a) If we produce 10 units of each product, how much is used up in the production process, and thus how much do we actually end up with?

(b) How many units of each product should be produced in order to end up with 10 units of each product?
3. For each of the following systems of equations, one or two of the coefficients and/or right-hand-side number is missing (where there is a “?”). Circle one of the three possibilities (true, don’t know, or false) for each of the three possible number of solutions (0, 1 or \( \infty \) solutions). The “might be true” means “might be true, depending what the value of the unknown ? values are.”

<table>
<thead>
<tr>
<th>Example:</th>
<th>Must be true</th>
<th>Might be true</th>
<th>Must be false</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + y = 1 )</td>
<td>Has no solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( x + y = 2 )</td>
<td>Has exactly 1 solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td></td>
<td>Has ( \infty ) solutions.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 2x + 3y = 4 )</td>
<td>Has no solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 4x + ? y = 8 )</td>
<td>Has exactly 1 solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td></td>
<td>Has ( \infty ) solutions.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 2x + 3y = 4 )</td>
<td>Has no solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 4x + 6y = ? )</td>
<td>Has exactly 1 solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td></td>
<td>Has ( \infty ) solutions.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 2x + 3y = 4 )</td>
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</tr>
<tr>
<td>( 4x + ? y = ? )</td>
<td>Has exactly 1 solution.</td>
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<td></td>
<td>Has ( \infty ) solutions.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 2x + 3y = ? )</td>
<td>Has no solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td>( 4x + 5y = ? )</td>
<td>Has exactly 1 solution.</td>
<td>Must be true</td>
<td>Might be true</td>
</tr>
<tr>
<td></td>
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</tr>
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</table>
4. A couple of kids from your neighborhood, Ronny and Nancy put on a pet show last week, and charged for admission: $5 per kid and $10 per adult. If they sold a total of 25 tickets and if they took in a total of $165, how many tickets did they sell to kids and how many to adults? Solve this problem by setting up a system of equations (two equations, two unknowns) and solving it by finding the inverse of the 2 x 2 matrix (rather than by Gaussian elimination or by guessing).
Each of the following is the final matrix of a Gaussian elimination process. Give the solutions to the corresponding systems of linear equations. You can use $x$ and $y$ (and $z$, if there are three variables) as the variables.

<table>
<thead>
<tr>
<th>Final matrix</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 1 \ 0 &amp; 1 &amp; 0 \end{bmatrix}$</td>
<td></td>
</tr>
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<td>$\begin{bmatrix} 1 &amp; 0 &amp; 1 &amp; 1 \ 0 &amp; 1 &amp; 1 &amp; 0 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 2 &amp; 2 \ 0 &amp; 0 &amp; 0 &amp; 3 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 2 \ 0 &amp; 1 &amp; 3 \ 0 &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>
6. Solve the following system of equations by doing Gaussian elimination on the corresponding augmented matrix:

\[
\begin{align*}
x + y + z &= 2 \\
2x - y + z &= 7 \\
-3x + 2y - 2z &= -13
\end{align*}
\]