DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use one sheet (one side of paper) of notes, but no other materials or resources (such as a calculator, class notes or old HW).

There is no sharing with a friend or neighbor.

For all problems you must show pertinent work.
1. Suppose that you want to build a fence against a wall such that there are three separate areas fenced in, as shown below.

As you can see, we need fencing everywhere except along the wall. We need each pen to have an area of exactly 100 square feet, for a total of 300 square feet, since there are three pens. Using the **Lagrange multiplier method**, find the values of $x$ and $y$ that would minimize the amount of fencing needed. All of the fencing costs the same amount. (If you happen to find the correct values of $x$ and $y$ by any other means than the Lagrange multiplier method, you will get some points, but not very many.)
2. Suppose \( f(x, y) = e^{xy} \).

(a) Find \( \frac{\partial f}{\partial x} \), \( \frac{\partial^2 f}{\partial x^2} \), and \( \frac{\partial^3 f}{\partial x^3} \), and use these to guess/predict what \( \frac{\partial^n f}{\partial x^n} \) would be.

(b) Evaluate each of these four derivatives found in (a) at the point \((x, y) = (1, 2)\). (Write your answers in terms of \( e \), since you don’t have a calculator.)

3. Where \( f(x, y, z) = \frac{x \ln x \ y}{z} \), find:

(a) \( \frac{\partial f}{\partial x} \)

(b) \( \frac{\partial f}{\partial y} \)

(c) \( \frac{\partial f}{\partial z} \)
4. Suppose that in a certain country the daily demand $f(p_C, p_T)$ for coffee is a function of $p_C$, the price charged per cup of coffee, and of $p_T$, the price charged per cup of tea, which is coffee’s primary competition in this country.

(a) Would you expect that $\frac{\partial f}{\partial p_C}$ would be positive or negative or zero? Why?

(b) Would you expect that $\frac{\partial f}{\partial p_T}$ would be positive or negative or zero? Why?

(c) Suppose that you sell coffee for a living, so of course your profit depends both on demand for coffee and the price you can sell it for. Also, suppose now (regardless of your answer in (a)) that $\frac{\partial f}{\partial p_C} > 0$. What action would you take in regards to the price of coffee? Why?
5. We are interested in how much chocolate is purchased (in millions of pounds) each year in the United States as a function of the price of the chocolate (in dollars per pound). Suppose that based on data from the past several years, we have come up with a “best fit” model

\[ c = -2p + 14 \]

where \( c \) is the demand for chocolate (i.e. the amount purchased) and \( p \) is the price of chocolate.

(a) What do the \(-2\) and the 14 in the above equation represent/mean/describe?

(b) According to this model, approximately how much chocolate would be purchased if the price were $2.50 per pound?

According to this model, approximately how much chocolate would be purchased if the price were $7.00 per pound? Comment on this level of demand.

(c) At approximately what price would there be a demand for 8 million pounds of chocolate per year?

What would the price need to be in order for there to be a demand of 16 million pounds of chocolate per year? Comment on this price.
6. Given the function \( f(x, y) = 3x^2 - 6xy - 9y + y^3 \), by using first and second derivatives, find the critical point(s) for this function, and determine what type of point (max, min, etc.) each point is.