Math 215 Fall 2009 Exam 4 Solutions

Low: xx   Average: xx   High: 100
Approximate grade breakdown (“if I curved this exam”):

90 – 100   A
80 – 90   B
0 – 80   C or below

1. \[ \frac{1 \cdot \frac{2}{3}}{3} = \frac{2}{9} \]

\[ \frac{\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{3}{3}}{3} = \frac{5}{9} \]

\[ \frac{2}{9} = \frac{2}{6} \]

\[ \frac{\frac{1}{3} \cdot \frac{0}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{3}{3}}{2} = \frac{8}{12} = \frac{8}{12} \]

2. \[ \frac{1+6}{6^6} = \frac{7}{6^6} \]

\[ 1 - \frac{7}{6^6} \]

\[ \left(\frac{3}{6}\right)^6 \]

\[ 1 - \left(\frac{5}{6}\right)^6 \]

\[ \frac{6!}{6^6} \text{ or } \frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} \]

\[ \frac{6}{6^6} \text{ or equivalently } \frac{6! \cdot 1 \cdot 1 \cdot 1 \cdot 1}{6^6} \]

3.

<table>
<thead>
<tr>
<th>Probability</th>
<th>If no testing is done</th>
<th>If test is positive</th>
<th>If test is negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has flu</td>
<td>.05</td>
<td>.62252</td>
<td>.00324</td>
</tr>
<tr>
<td>Does not have flu</td>
<td>.95</td>
<td>.37748</td>
<td>.99676</td>
</tr>
</tbody>
</table>

\[ \Pr(F \mid +) = \frac{\Pr(F) \Pr(\overline{F} \mid F)}{\Pr(\overline{F} \mid +)} = \frac{(0.05)(0.94)}{(0.05)(0.94) + (0.95)(0.03)} = 0.62252 \]

\[ \Pr(F' \mid +) = \frac{\Pr(F') \Pr(\overline{F} \mid F')}{\Pr(\overline{F} \mid +)} = \frac{(0.95)(0.03)}{(0.05)(0.94) + (0.95)(0.03)} = 0.37748 \]

or simply find one of these two and subtract it from 1 to get the other.

\[ \Pr(F \mid -) = \frac{\Pr(F) \Pr(\overline{F} \mid F)}{\Pr(\overline{F} \mid -)} = \frac{(0.05)(0.06)}{(0.05)(0.06) + (0.95)(0.97)} = 0.00324 \]

\[ \Pr(F' \mid -) = \frac{\Pr(F') \Pr(\overline{F} \mid F')}{\Pr(\overline{F} \mid -)} = \frac{(0.95)(0.97)}{(0.05)(0.06) + (0.95)(0.97)} = 0.99676 \]

or simply find one of these two and subtract it from 1 to get the other.
4. \(\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F) = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6}\)

\(\Pr(E') = 1 - \Pr(E) = 1 - \frac{2}{6} = \frac{4}{6}\)

\(\Pr(F') = 1 - \Pr(F) = 1 - \frac{3}{6} = \frac{3}{6}\)

\(\Pr(E' \cap F') = \Pr((E \cup F)') = 1 - \Pr(E \cup F) = 1 - \frac{4}{6} = \frac{2}{6}\)

\(\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{1/6}{3/6} = \frac{1}{3}\)

\(\Pr(F|E) = \frac{\Pr(E \cap F)}{\Pr(E)} = \frac{1/6}{2/6} = \frac{1}{2}\)

\(E\) and \(F\) are independent, as is proven by any one of the three facts which you can see are true from the work you did: \(\Pr(E|F) = \Pr(E), \Pr(F|E) = \Pr(F), \) or \(\Pr(E \cap F) = \Pr(E) \cdot \Pr(F).\)

If \(E\) and \(F\) would be mutually exclusive if \(E \cap F = \emptyset\) or equivalently \(\Pr(E \cap F) = 0.\) Since \(\Pr(E \cap F) = \frac{1}{6}, \) \(E\) and \(F\) are not mutually exclusive.

5. \((.98)(.97)(.93)^2 = .8222\)

6. \(\frac{26}{51}, \) which you could figure out simply by realizing that if your first car is red, then of the remaining 51 cards, 26 of them are black, since no black card has been chosen yet, or you could use the formula \(\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{\Pr(E) \Pr(F|E)}{\Pr(F)} = \frac{26 \cdot 26}{51 \cdot 52} = \frac{26}{51}.\)

There are two ways for event \(E\) to occur: the first card might be red or it might be black, so \(\Pr(E) = \Pr(E \cap F) + \Pr(E \cap F') = \Pr(F) \Pr(E|F) + \Pr(F') \Pr(E|F') \)

\(= \frac{26}{52} \cdot \frac{26}{51} + \frac{26}{52} \cdot \frac{25}{51} = \frac{26(25+26)}{52(51)} = \frac{26}{52}.\)

The results that \(\Pr(E|F) \neq \Pr(E)\) mean that \(E\) and \(F\) are not independent.