Math 215 Fall 2009 Exam 2 Solutions
Low: 54   Average: 78   High: 98

Approximate grade breakdown (“if I curved this exam”):

- 90 – 100   A
- 80 – 90   B
- 0 – 80   C or below

1. This just like homework problem 7.5.14.

(a) At \( y = 1983 \), \( c = 1.234(1983) - 2336 \approx 111 \).

(b) \( 130 = 1.234y - 2336 \) leads to \( y = \frac{130 + 2336}{1.234} \approx 1998.38 \), so part way through 1998.

(c) 1.234 tells us that for each unit increase in \( y \) (that is, with each passing year), the number of cars on the road will increase by 1.234 million. –2336 is the number of cars (according to this model) on the road in the year 0, which of course doesn’t make sense, which is related to the fact that this model is not reasonable for really small values of \( y \) (e.g. 0, or really any values before around 1900, when cars were invented, for that matter), while the model is pretty good around the years for which we actually have data. In general, interpolation is much more accurate than extrapolation.

2. This homework problem 7.4.17—solution is in Student Solutions Manual.

3. This is example 7.3.4.

4. There are two ways to solve this: using the Lagrange Multiplier Method or not.

First, not: Solve for one variable in terms of the other in the constraint

\[ 60x^{3/4}y^{1/4} = 600, \text{ i.e. that } x^{3/4}y^{1/4} = 10 \text{ so that } y^{1/4} = \frac{10}{x^{3/4}}, \text{ i.e. that } y = \frac{10000}{x^3}. \]

Then the cost function becomes \( 100x + 200\left(\frac{10000}{x^3}\right) = 100x + 2000000x^{-3} \).

We find the derivative of this function and solve for it equal to 0,

\[ 100 - 3 \cdot 2000000x^{-4} = 0, \text{ that is } 100 - \frac{6000000}{x^4} = 0, \]

so that \( x^4 = 6000000 \), so \( x = 15.65 \), so \( y = \frac{10000}{15.65^3} = 2.61 \).

Now, with: The combined function (objective + \( \lambda \cdot \text{constraint} \)) is

\[ F(x, y, \lambda) = 100x + 200y + \lambda(60x^{3/2}y^{1/2} - 600) \]

which has partial derivatives of
Setting the first two $= 0$ and solving for lambda, then setting these equal to each other:

\[
\frac{-100x^{3/4}}{45y^{1/4}} = \frac{-200y^{3/4}}{15x^{3/4}},
\]

which we cross multiply to get $-1500x = -9000y$, so $y = \frac{x}{6}$, which we substitute into the final partial derivative

\[
60x^{3/4}y^{1/4} - 600 = 0, \text{ i.e. } x^{3/4}y^{1/4} = 10, \text{ i.e. } x^3y = 10000
\]

to get $x = 10000$, so that $x^4 = 60000$ from which

\[
x = 15.65 \text{ and } y = \frac{15.65}{6} = 2.61
\]

(b) $600 = 60x^{3/4}y^{1/4}$, i.e. $10 = x^{3/4}y^{1/4}$,

i.e. $10000 = x^3y$, so $y = \frac{10000}{x^3}$, which is shown at right along with the point found above. This represents all of the values of $x$ and $y$ which would result in a production level of 600.

5. \[
\frac{\partial f}{\partial x} = ye^{xy}, \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} ye^{xy} = 1 \cdot e^{xy} + y \cdot xe^{xy} = e^{xy} + xye^{xy}.
\]

\[
\frac{\partial f}{\partial y} = xe^{xy}, \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} xe^{xy} = 1 \cdot e^{xy} + x \cdot ye^{xy} = e^{xy} + xye^{xy}.
\]

\[
\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} ye^{xy} = y \cdot ye^{xy} = y^2 e^{xy}.
\]

Similarly, \[
\frac{\partial^n f}{\partial x^n} = x^n e^{xy}.
\]

6. $f(m, p, r) = 2.186m^{595}r^{922}p^{-543}$, so

\[
\frac{\partial f}{\partial m} = 2.186(595)m^{1-595}r^{922}p^{-543} > 0, \text{ i.e. food consumption would increase if income increases.}
\]

\[
\frac{\partial f}{\partial r} = 2.186(922)m^{595}r^{1-922}p^{-543} > 0, \text{ i.e. food consumption would increase if the price of other goods increases (people buy what is cheaper).}
\]

\[
\frac{\partial f}{\partial p} = 2.186(-543)m^{595}r^{922}p^{-543-1} < 0, \text{ i.e. food consumption would decrease if the price of food increases.}
\]