1. \((R \cup S \cup T) \cap (R' \cup S' \cup T') = (R \cup S \cup T) \cap (R \cap S \cap T)',\)
i.e. the union of all three, but not in the intersection of the three.

\[(R \cap T) \cup (S \cap R' \cap T')\]

From the given information, we find that

\[n(\text{Swim}) = \text{Total persons} - n(\text{Don't swim}) = 100 - 30 = 70,\]

so that the number who like to only swim is 50 (since 20 like to do both). So we have the following diagram, from which we see there is no one who only likes to surf.
For some of these four, using DeMorgan’s Laws helps, and for some, drawing Venn Diagrams helps.

\[(A \cup B)' = A \cap B'\]

\[((A \cup B) \cap C') = (A' \cup B) \cup C = (A \cap B') \cup C\]

\[(A \cup B' \cup C') \cap (A \cup B' \cup C') = \emptyset\] (the empty set), since \(S \cup S' = \emptyset\) for any set \(S\).

\[(A \cup B \cup A') = U = \emptyset, \text{ since } (A \cup A') = U, \text{ the universal set.}\]

2. **First page of questions**

3 \cdot 8 \cdot 9

\[C(7,3)\]

In how many 7-digit numbers (i.e. 1000000 – 9999999) are there in which exactly 2 of the digits are the same?

# ways to choose which 2 digits \* # ways to choose the 6 digits that will be in the 7 digit number = \[C(7,2) \cdot 9 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5\] (the first 9 is choosing the very first digit from 1 to 9, and the second 9 is then choosing the first of the other 5 digits, which can come from 0 to 9, except for the number chosen for the first digit).

Simplest approach (you could also try to break it down into the various ways this actually happens, which is somewhat complicated):

\[\# \text{ ways at least one digit is different } = \text{ total } \# - \# \text{ not at least one different}\]

= total \# - \# all digits same

= \[9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 - 9 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1\]

\[= 9000000 - 9\]

\[2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7\] (each 2 corresponds to the two choices for each friend: invite or don’t invite), which is the same as \(C(7,0) + C(7,1) + \cdots C(7,7)\).

14!

Since all four need to get (at least) 1 copy, the only question is who is going to get the other two. There are two ways for this to happen: one person gets both, and there are 14 ways to choose who that one person would be; or two persons each get one, and there are \(C(14,2)\) ways to choose who those two would be. So there are a total of \(14 + C(14,2)\) ways to do this.
Second page of questions

\(C(52,5)\)

\(# \text{ ways to choose the } 2 \text{ Aces from } 4 \times \text{ # ways to choose the other } 3 \text{ cards from the } 48 \text{ non-Ace cards} = C(4,2) \cdot C(48,3)\)

\(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4\) (you simply need to choose which 3 from the four suits, which 4 from the four suits, …, and which 7 from the four suits)

\(# \text{ different pairs of numbers} \times \text{ number of ways to choose which is } 2 \text{ and which is } 3 \times \text{ # ways to choose the } 2 \text{ from the } 4 \times \text{ # ways to choose the } 3 \text{ from the } 4 = C(13,2) \cdot 2 \cdot C(4,2) \cdot C(4,3)\)

\(10 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10\)

\(10 \cdot 26 \cdot 25 \cdot 24 \cdot 9 \cdot 8 \cdot 7\)

\(# \text{ ways to choose which } 2 \text{ numbers are the same} \times \text{ # ways to choose which 2 letters are the same} \times \text{ # ways to choose the numbers} \times \text{ # ways to choose the letters} = C(4,2) \cdot C(3,2) \cdot (10 \cdot 9 \cdot 8) \cdot (26 \cdot 25)\). It turns out this is almost right (but that it is a bit more complicated than this—there is some repeat counting), so gave full credit for answers that are slight variations of this, but most persons ended up getting partial credit. Note: the answer 10 \cdot 26 \cdot 1 \cdot 25 \cdot 1 \cdot 9 \cdot 8 would be correct if we assume the first two of the four numbers are the same the first two of the three letters are the same.

\(# \text{ ways to choose which } 4 \text{ of the } 7 \text{ positions would be numbers} (\text{which } 3 \text{ would be letters would be whatever is left over}) \times \text{ # ways to choose the digits and letters} = C(7,4) \cdot 10^4 \cdot 26^3\). It turns out this is almost right (but that it is a bit more complicated than this—there is some repeat counting), so gave full credit for answers that are slight variations of this, but most persons ended up getting partial credit.
$4 \cdot 5 = 20$

# ways to choose the 2 boys * # ways to choose the 2 girls * # ways to pair them up * # ways to decide who is first and who is second couple

$$= C(4,2) \cdot C(5,2) \cdot 2 \cdot 2 = \frac{4 \cdot 3}{2} \cdot \frac{5 \cdot 4}{2} \cdot 2 \cdot 2 = 240,$$

or simply $(4 \cdot 3)(5 \cdot 4)$, the number of ways to choose the two boys (order matters) * the number of ways to choose the two girls (order matters)

$$C(9,4) = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!} = \frac{9 \cdot 7 \cdot 2}{4 \cdot 3} = 63 \cdot 2 = 126.$$

# ways to decide whether girls or boys are first * # ways to order boys * # ways to order the girls

$$P(5,3) = 5 \cdot 4 \cdot 3 = 60,$$

or $5 \cdot 8 \cdot 7$ if the VP and secretary are not required to be female (the question was a bit ambiguous about that)