1. \[
\begin{align*}
\frac{20}{36} + \frac{15}{36} - \frac{7}{36} &= \frac{28}{36} = \frac{7}{9} \\
\frac{7}{36} &= \frac{7}{15} \\
\frac{20}{36} &= \frac{5}{9} \\
\end{align*}
\]
Whatever \( E \) is, \( \Pr(E \cup E') = 1 \) (since there is 100% certainty that either \( E \) occurs or that it doesn’t), and \( \Pr(E \cap E') = 0 \) (it cannot be that \( E \) occurs and that it does not occur).

2. \[
\frac{4 \cdot 3 \cdot 3}{7 \cdot 6 \cdot 5} = \frac{6}{35}
\]
\[
\frac{C(4,2)C(3,1)}{C(7,3)} = \frac{18}{35}
\]

3. \( \Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F) = \frac{4}{6} \)

\( \Pr(E') = 1 - \Pr(E) = \frac{4}{6} \)

\( \Pr(F') = 1 - \Pr(F) = \frac{3}{6} \)

\( \Pr(E' \cap F') = 1 - \Pr((E' \cap F')') = 1 - \Pr(E \cup F) = \frac{2}{6} \)

\( \Pr(E \mid F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{1/6}{3/6} = \frac{1}{3} \)

\( \Pr(F \mid E) = \frac{\Pr(E \cap F)}{\Pr(E)} = \frac{1/6}{2/6} = \frac{1}{2} \)

\( \Pr(E' \mid F) = 1 - \Pr(E \mid F) = \frac{2}{3} \) or \( \Pr(E' \mid F) = \frac{\Pr(E' \cap F)}{\Pr(F)} = \frac{2/6}{3/6} = \frac{2}{3} \)

\( \Pr(F' \mid E') = \frac{\Pr(E' \cap F')}{\Pr(E')} = \frac{2/6}{4/6} = \frac{1}{2} \)

Yes. You can see this in a few ways:

\( \Pr(E \cap F) = \Pr(E) \Pr(F) \), \( \Pr(E \mid F) = \Pr(E) \), and \( \Pr(F \mid E) = \Pr(F) \)

No, \( E \) and \( F \) are not mutually exclusive. Mutually exclusive means \( \Pr(E \cap F) = 0 \), i.e. that both \( E \) and \( F \) cannot occurs simultaneously, which is not the case here. You could also point out that \( \Pr(E \cup F) \neq \Pr(E) + \Pr(F) \).
4. Where F is “has flu” and + is “tests positive,” from the given information:

\[
\begin{align*}
\Pr(F) &= .10 \\
\Pr(F') &= 1 - \Pr(F) = 1 - .10 = .90 \\
\Pr(+ | F) &= .96 \\
\Pr(- | F) &= 1 - \Pr(+ | F) = 1 - .96 = .04 \\
\Pr(+ | F') &= 1 - \Pr(- | F') = 1 - .95 = .05 \\
\Pr(- | F') &= .95
\end{align*}
\]

We first find that

\[
\Pr(+) = \Pr((+ \cap F) \cup (+ \cap F')) = \Pr(+ \cap F) + \Pr(+ \cap F') \\
= \Pr(F) \Pr(+ | F) + \Pr(F') \Pr(+ | F') = (.10)(.96) + (.90)(.05) = .1410
\]

and consequently \( \Pr(-) = 1 - \Pr(+) = 1 - .1410 = .8590 \), or you could (unnecessarily) do all the work to find

\[
\Pr(-) = \Pr((- \cap F) \cup (- \cap F')) = \Pr(- \cap F) + \Pr(- \cap F') \\
= \Pr(F) \Pr(- | F) + \Pr(F') \Pr(- | F') = (.10)(.04) + (.90)(.95) = .8590
\]

Next, if you test positive, then

\[
\Pr(F | +) = \frac{\Pr(F \cap +)}{\Pr(+)} = \frac{\Pr(F \cap +)}{\Pr(+)} = \frac{.10(0.96)}{.1410} = .6809,
\]

and consequently

\[
\Pr(F' | +) = 1 - \Pr(F | +) = 1 - .6809 = .3191
\]

or you could have done all the work to find that

\[
\Pr(F' | +) = \frac{\Pr(F' \cap +)}{\Pr(+)} = \frac{\Pr(F') \Pr(+ | F')}{\Pr(+)} = \frac{.90(0.05)}{.1410} = .3191
\]

Finally, if you test negative, then

\[
\Pr(F | -) = \frac{\Pr(F \cap -)}{\Pr(-)} = \frac{\Pr(F \cap -)}{\Pr(-)} = \frac{.10(0.04)}{.8590} = .0047,
\]

and \( \Pr(F' | -) = 1 - \Pr(F' | +) = 1 - .0047 = .9953 \)

or you could have done

\[
\Pr(F' | -) = \frac{\Pr(F' \cap -)}{\Pr(-)} = \frac{\Pr(F') \Pr(- | F')}{\Pr(-)} = \frac{.90(0.95)}{.8590} = .9953
\]