DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.
You may use one page (one side) of notes and a calculator, but no other materials or resources (such as notes, old HW, etc.).
There is no sharing with a friend or neighbor.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

Frank and Ernest

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A diabolical new testing technique: math essay questions.
1. Find the inverse, if it exists, of \( A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \).

2. Each of the following is the final matrix of a Gaussian elimination process. Give the solutions to the corresponding systems of linear equations. You can use \( x \) and \( y \) (and \( z \), if there are three variables) as the variables.

<table>
<thead>
<tr>
<th>Final matrix</th>
<th>Solution</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\] |          |
| \[
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\] |          |
| \[
\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}
\] |          |
| \[
\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}
\] |          |
| \[
\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}
\] |          |
3 Each of the following is the final matrix of a Gaussian elimination process. Give the 
solutions to the corresponding systems of linear equations. You can use $x$ and $y$ (and $z$, 
if there are three variables) as the variables. If $A + B$ and/or $AB$ are/is not defined, just 
write that instead of a matrix size.

<table>
<thead>
<tr>
<th>Size of A</th>
<th>Size of AB</th>
<th>Size of A + B</th>
<th>Size of AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x 4</td>
<td>4 x 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 x 4</td>
<td>3 x 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 2</td>
<td>2 x 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 3</td>
<td>3 x 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 x 2</td>
<td>3 x 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. A company produces two products, but uses up some of each product in the production 
process. Suppose the input-output matrix is $A = \begin{bmatrix} .1 & .8 \\ .4 & .2 \end{bmatrix}$.

(a) Suppose the company produces 10 units of each product. How much of each 
product is used up in the production process?

(b) Suppose we want to end up with 10 units of each product. How much should the 
company produce of each product?
5. We are interested in solving the following system of equations,

\[2x + 3y = a\]
\[4x + by = 8\]

where \(a\) and \(b\) are some constants (whose values haven’t yet been decided). Give an example of values of \(a\) and \(b\) that result in the system having:

No solution: \(a = \quad b = \)

One solution: \(a = \quad b = \)

Infinite solutions: \(a = \quad b = \)

6. Given the matrix

\[
A = \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

find \(A^2\) (i.e., \(AA\)), and use this result to find the solution to \(AX = B\), where \(B = \begin{bmatrix} 10 \\ -4 \\ 0 \\ -2 \end{bmatrix} \).
7. A college student wishes to invest $1000 in three sources: bonds paying 5% annually, CDs paying 4%, and high risk mortgages paying 10%. The student wants an annual income of $50. Also, the bank employee making this investment decision wants to put the same amount into the CDs (which are less risky) as into the bonds and mortgages combined. How much should be invested in the three sources? Solve this problem by setting up a system of equations (three equations, three unknowns) and solving it using Gaussian elimination.