DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use one sheet (one side of paper) of notes and a calculator, but no other materials or resources (such as class notes or old HW).

There is no sharing with a friend or neighbor.

Only your exam, sheet of notes and calculator may be out.

For all problems you must show pertinent work.
1. Suppose that you want to build a fence against a wall such that there are two separate areas fenced in, as shown below.

As you can see, we need fencing everywhere except along the wall. Suppose we need each pen to have an area of exactly 100 square feet. Using the Lagrange multiplier method, find the values of $x$ and $y$ that would minimize the amount of fencing needed.
2. The following table gives the percentage of persons 25 years and over who have completed four or more years of college in recent years:

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>10.7</td>
</tr>
<tr>
<td>1975</td>
<td>13.9</td>
</tr>
<tr>
<td>1980</td>
<td>16.2</td>
</tr>
<tr>
<td>1985</td>
<td>19.4</td>
</tr>
<tr>
<td>1990</td>
<td>21.3</td>
</tr>
<tr>
<td>1995</td>
<td>23.0</td>
</tr>
</tbody>
</table>

For this data, we can find the least squares ("best fit") line $p = 0.497t + 11.2$, where $t$ is the number of years after 1970 (so, for example, the year 1977 would correspond to the value of $t = 7$).

(a) This information is gathered only every five years, but we are interested in the percentage for the year 1993. Using the least squares line just given, estimate the percentage for the year 1993.

(b) Assuming this current trend continues (i.e. using the least squares line given above), estimate in what the percentage will reach 30.

(c) What estimate would you come up with for the percentage in the year 1945? Comments/observations on this estimate?
3. Suppose that in a certain country the daily demand \( f(p_1, p_2) \) for coffee is a function of \( p_1 \), the price of tea, and of \( p_2 \), the price of coffee. (Assume coffee and tea are competing products, so people will tend to choose one or the other.)

(a) Would you expect that \( \frac{\partial f}{\partial p_1} \) would be positive or negative or zero? Why?

(b) Would you expect that \( \frac{\partial f}{\partial p_2} \) would be positive or negative or zero? Why?

(c) Suppose that you sell coffee for a living, so of course your profit depends both on demand for coffee and the price you can sell it for. Also, suppose now (regardless of your answer in (b)) that \( \frac{\partial f}{\partial p_2} > 0 \). What action would you take in regards to the price of coffee? Why?

4. Continuing what we started in Problem 3. Suppose now we have a particular function for \( f \): \( f(p_1, p_2) = 8 \frac{p_1}{p_2} \), where \( f \) is in thousands of pounds of coffee per day, and \( p_1 \) and \( p_2 \) are in dollars per pound of tea and coffee, respectively.

(a) Compute \( f(4,2) \) and interpret this value.

(b) Find \( \frac{\partial f}{\partial p_1} \), then compute \( \frac{\partial f(4,2)}{\partial p_1} \) and interpret this value.

(c) Find \( \frac{\partial f}{\partial p_2} \), then compute \( \frac{\partial f(4,2)}{\partial p_2} \) and interpret this value.
Given the function \( f(x, y) = x^3 - 9x - 6xy + 3y^2 \), by using first and second derivatives, find the critical point(s) for this function, and determine what type of point (max, min, etc.) each point is.