1. \( A(100,3) = 100 \cdot 1.07^3 = 122.50 \), so after three years my $100 has grown to $122.50.

2. (a) \( \frac{\partial f}{\partial x} = 2x - 6 \); this describes how quickly \( f \) changes as \( x \) increases by one unit (and \( y \) is held constant).

\( \frac{\partial f}{\partial y} = -3y^2 + 12 \); this describes how quickly \( f \) changes as \( y \) increases by one unit (and \( x \) is held constant).

\( \frac{\partial f}{\partial x} (3,1) = 2 \cdot 3 - 6 = 0 \) and \( \frac{\partial f}{\partial y} (3,1) = -3 \cdot 1^2 + 12 = 9 \).

(b) \( \frac{\partial^2 f}{\partial x^2} = 2 \), \( \frac{\partial^2 f}{\partial y^2} = -6y \), \( \frac{\partial^2 f}{\partial x \partial y} = 0 \) and \( \frac{\partial^2 f}{\partial y \partial x} = 0 \).

(c) \( \frac{\partial f}{\partial x} = 2x - 6 = 0 \) where \( x = 3 \), and \( \frac{\partial f}{\partial y} = -3y^2 + 12 = 0 \) where \( y = \pm 2 \). So there are two critical points: (3,2) and (3,−2). We find that \( D(x, y) = 2 \cdot (-6y) - 0^2 = -12y \).

Since \( D(3,2) = -12 \cdot 2 = -24 < 0 \), we have a saddlepoint (neither max nor min) at (3,2). Just to remind you, this case of “neither” is quite different from the fourth case of “don’t know”, which occurs when \( D(x, y) = 0 \).

Since \( D(3,−2) = -12 \cdot (-2) = 24 > 0 \), we have either a max or a min. Since \( \frac{\partial^2 f}{\partial x^2} = 2 > 0 \), then we have a min at (3,−2).

3. (a) We would expect \( \frac{\partial t}{\partial e} > 0 \) since we would expect to produce more toasters (i.e. \( t \) would increase) if we hired more employees (i.e. as \( e \) increases).

(b) Similarly, we would expect \( \frac{\partial t}{\partial m} > 0 \).
4. (a) We would expect the $x$ side to be longer, since it is less expensive, and since each side contributes equally to making the area $xy$ larger. Because I expect $x$ to be larger than $y$, I draw my picture the way I did, not the other way around. That is, I didn’t just randomly draw my picture and then say, “Oh look, in my picture side $x$ seems larger than $y$, so it must be that $x$ is larger than $y$.”

(b) We are trying to minimize the amount we will spend on fencing. Since the $x$ side costs $4$ per foot and the $y$ side costs $6$ per foot, and since we only want one side of each (as seen in the picture), then our objective function is $f(x, y) = 4x + 6y$. The restriction on our solving this problem is that the area be 600 square feet: $xy = 600$, that is, $xy - 600 = 0$, so our constraint equation is $g(x, y) = xy - 600$. So our combined function is 

$$F(x, y, \lambda) = f(x, y) + \lambda g(x, y) = 4x + 6y + \lambda(xy - 600) = 4x + 6y + \lambda xy - 600\lambda.$$ 

We find first derivatives and set them equal to 0:

$$\frac{\partial F}{\partial x} = 4 + \lambda y, \text{ so that } \lambda = -\frac{4}{y}$$

$$\frac{\partial F}{\partial y} = 6 + \lambda x = 0, \text{ so that } \lambda = -\frac{6}{x}, \text{ which from the first equation leads to } -\frac{6}{x} = -\frac{4}{y}, \text{ so that } 6y = 4x, \text{ so that } y = \frac{2}{3} x \text{ (or } x = \frac{3}{2} y, \text{ if you prefer}).$$

$$\frac{\partial F}{\partial \lambda} = xy - 600 = 0, \text{ so where } y = \frac{2}{3} x \text{ we found using the first and second equations, this leads to } x(\frac{2}{3} x) = 600, \text{ so } x^2 = 900, \text{ so } x = 30 \text{ and } y = \frac{2}{3}(30) = 20.$$ 

And notice that $x$ is larger than $y$, as we expected.

If you happened to have mixed up the objective and constraint functions, then you would have ended up with something like $F(x, y, \lambda) = xy + \lambda(4x + 6y - 600)$, which would have led to $x = 75$ and $y = 50$.

5. (a) At this price we would have $y = -1366(2.00) + 12842 = 10,100$ miles driven per car.

(b) $-1366$ is the amount by which the number of miles driven will change per year per ont unit (in this case, $1.00$) increase in the cost of gas. In other words, if gas goes up by one dollar, the average car will be driven 1366 miles less per year.

12842 is the amount people would drive if gas were to cost $0$, that is, if it were free.

(c) We are solving for $x$ in $0 = -1366x + 12842$, so that the necessary price would be

$$1366x = \frac{12842}{1366} = 9.40. \text{ We don’t believe this would actually happen, as certain people will need to drive no matter what the cost of gas is, so that it will never actually occur that the average car would drive 0 miles per year. Also, the laws of supply and demand tell us that the price of gasoline would never rise to the point that no one wants to buy it. So our model is probably not very useful for predicting the number of miles driven when gas is expensive (i.e. when } x \text{ is large, such as $8$ or $9$ or more per gallon).}$$